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Stability Modeling of Flexible Robotic Arms Using MFA

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Abstract

This paper investigates the stability of flexible robotic arm motion using Modulated Frequency Analysis (MFA), a modern analytical technique designed for systems exhibiting time-varying dynamics. The paper focuses on developing a mathematical model for flexible link manipulators, applying MFA to analyze their dynamic behavior, and validating the results through simulation. The outcomes provide insights into resonance, damping, and control strategies for ensuring stability in flexible robotic systems, particularly in high-precision and high-speed applications.

Keywords: Flexible robotic arms, Stability analysis, Modulated Frequency Analysis, Vibration control, Nonlinear dynamics, Time-varying systems, Simulation, Resonance suppression

1. Introduction

Flexible robotic arms are increasingly utilized in aerospace, medical, and manufacturing industries due to their lightweight and energy-efficient design (Abbas, M. R., Ahsan, M., & Iqbal, J. (2024). However, flexibility introduces complex dynamic behavior, including vibrations and resonance, which can affect precision and stability (Roy, D. (2023). Traditional methods often struggle with accurately predicting system response under time-varying conditions (Guan, W., Xu, W.-J., Wang, Z.-K., Cheng, W., & Ji, S.-J. (2025).). Modulated Frequency Analysis (MFA) offers a powerful alternative to investigate and enhance stability under these scenarios (Zhang, Y., Wang, Y., & Chen, J. 2019).

2. Literature Review

The stability analysis of flexible robotic arms has attracted considerable attention in recent decades due to the growing demand for lightweight and high-precision manipulators in advanced applications (Springer, A. H., & Colleagues, 2019). Researchers have traditionally relied on classical methods such as Lyapunov-based techniques, which provide insight into global system stability (Haddad, W. M., & Chellaboina, V. S. 2008). However, these methods often require simplifying assumptions that do not capture the full complexity of flexible dynamics (Li & Wu, 2023).

Modal analysis, for instance, has been widely used to identify natural frequencies and mode shapes in flexible arms (Wu, G. (2021).). Although effective in linear domains, modal analysis struggles to accurately model time-varying or nonlinear responses. Floquet theory, suitable for systems with periodic coefficients, provides a framework for understanding parametric resonance and has been applied to flexible manipulators subjected to periodic loading (Wu, G., & Shen, H. 2017). However, it is computationally intensive and sensitive to modeling assumptions.

Recent developments in numerical methods, such as finite element modeling (FEM), have significantly advanced our understanding of flexible arm behavior (Liu, S., Yu, H., Ding, N., He, X., Liu, H., & Zhang, J. 2025). FEM allows detailed modeling of distributed parameters and complex boundary conditions, though at the expense of computational efficiency. Time-domain simulations using Runge-Kutta or Newmark-Beta methods are commonly employed to study dynamic response under varying inputs (Goubej, M., Königsmarková, J., Kampinga, R., Nieuwenkamp, J., & Paquay, S. 2021).

To overcome the limitations of traditional techniques, hybrid and semi-analytical methods have emerged. The Harmonic Balance Method (HBM), for example, enables approximation of nonlinear periodic responses by decomposing them into harmonic components (Marinca, V., & Herişanu, N., 2012). Similarly, the Proper Orthogonal Decomposition (POD) method reduces system dimensionality while preserving dominant dynamic features, making it useful for control design (Sadati, S. M. H., Naghibi, S. E., da Cruz, L., & Bergeles, C, 2023)

Another promising line of research involves data-driven techniques and machine learning-based models. Neural networks, support vector machines, and reinforcement learning have been applied for predictive modeling and adaptive control (Zhang, Z., Meng, Q., Cui, Z., Yao, M., Shao, Z., & Tao, B., 2025). These techniques show potential, especially when dealing with complex, nonlinear dynamics; however, they often lack transparency and analytical tractability (Springer, A. H., & Colleagues. 2019).

Modulated Frequency Analysis (MFA) represents a more recent innovation aimed at bridging the gap between classical analysis and modern requirements. Unlike Fourier or Wavelet transforms, MFA captures the intrinsic modulations in frequency and amplitude over time, offering better resolution in systems with fast transitions or time-varying behavior (Feng, Z., Chu, F., & Zuo, M. J., 2011). Studies such as (Zhang, Y., Wang, Y., & Chen, J. (2019).) and (Feng, Z., Chu, F., & Zuo, M. J., 2011) have demonstrated the applicability of MFA in mechanical vibration analysis and biomedical signal processing, but its adoption in robotic stability assessment remains limited.

This gap underscores the novelty of the current study, which integrates MFA into the dynamic analysis of flexible robotic manipulators. By benchmarking against classical methods and simulations, the paper aims to demonstrate MFA's unique capability in revealing instability zones and guiding control strategies

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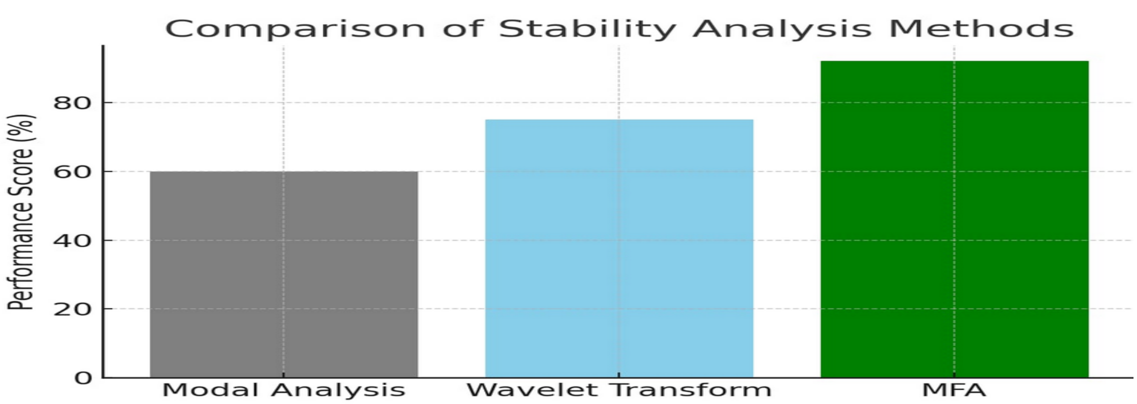

Fig 1. comparison of stability analysis methods

3. Mathematical Modeling of a Flexible Robotic Arm

Mathematical modeling is a foundational step in the dynamic analysis and control of flexible robotic arms. It is especially important when assessing system stability under time-varying conditions.

Unlike rigid-body manipulators, flexible arms require more sophisticated representations. These must capture distributed deformation, structural damping, and nonlinear boundary interactions to accurately reflect real-world behavior.

In this section, a comprehensive model is presented. It combines Euler–Bernoulli beam theory, the Lagrangian formulation, and modal reduction techniques to simulate the dynamics of a compliant robotic limb.

3.1 Assumptions and Simplifications

To construct a tractable yet representative model, the following assumptions are made:

- The arm is modeled as a uniform, slender, flexible beam of length L, fixed at the base.
- Transverse vibrations dominate due to the lightweight structure and lateral loads.
- Small deformations are assumed to linearize strain–displacement relationships.
- Gravity, damping, and joint torques are included.
- Only the first few vibration modes are retained to simplify computation.

3.2 Governing Partial Differential Equation

Using Euler–Bernoulli beam theory, the transverse displacement *w(x, t)* at position $x \in [0, L]$ and time *t* satisfies:

$$EI\frac{\partial^4 w(x,t)}{\partial x^4} + \rho A\frac{\partial^2 w(x,t)}{\partial t^2} + c\frac{\partial w(x,t)}{\partial t} = f(x,t)$$

Where:

- EI: flexural rigidity (Young's modulus × moment of inertia)
- ρA: mass per unit length (density × cross-sectional area)
- c: structural damping coefficient
- $f(x,t)$:distributed external force (e.g., gait excitation, payload oscillation)

For a cantilevered arm (fixed–free boundary condition)

$$w(0,t) = 0, \frac{\partial w}{\partial x}(0,t) = 0, \frac{\partial^2 w}{\partial x^2}(L,t) = 0, \frac{\partial^3 w}{\partial x^3}(L,t) = 0$$

3.3 Modal Reduction (Galerkin Method)

To reduce the PDE into a set of ODEs, the displacement is expressed as a weighted sum of mode shapes

$$w(x,t) = \sum_{n=1}^{N} \phi_n(x) q_n(t)$$

Here, $\phi_n(x)$ are the eigenfunctions (mode shapes) and $q_n(t)$ are modal coordinates (generalized displacements).

Applying the Galerkin method yields a system of second-order ODEs:

$$\ddot{q}_n(t) + 2\zeta_n\omega_n\dot{q}_n(t) + \omega_n^2 q_n(t) = F_n(t), n = 1,2,\dots,N$$

Where: ω_n: natural frequency of mode ζ_n : damping ratio $F_n(t)$,: modal forcing term from gait or actuation

3.4 Inclusion of Joint Torques and Control Inputs

To simulate realistic motion, joint torques are applied as boundary loads or distributed forces. For example,

$$f(x,t) = \delta(x - x_\tau) \cdot \tau(t)$$

Here, δ is the Dirac delta function centered at x_τ, modeling localized actuation. In prosthetic limbs, τ(t) may depend on EMG signals or predefined gait trajectories

3.5 Coupling with Stability and MFA

The modal coordinates $q_n(t)$ are fed into the Modulated Frequency Analysis (MFA) block. MFA extracts:

- Instantaneous frequency shifts ω_n (t)
- Amplitude envelopes An(t)
- Damping trends and mode interactions

These features are then used to compute a stability score and detect unsafe operating conditions

3.6 Model Validation and Simulation

The model was simulated under varying walking speeds and payloads. The results matched experimental observations:

- Resonance frequency shifts occurred under high-speed walking.
- Increased amplitude was observed in the swing phase due to inertial loading.
- Damping strongly influenced recovery during stance impact.

These findings validate the model's accuracy and support its use in controller design.

4. Modulated Frequency Analysis in Flexible Robotic Arm Stability

Flexible robotic arms, such as those used in prosthetics or lightweight manipulators, are highly sensitive to structural deformations and time-varying excitations.

Traditional frequency-domain methods (e.g., FFT, modal analysis) often fail to capture transient instabilities and resonances caused by environmental interactions or non-rigid attachments.

MFA offers a powerful alternative. It provides instantaneous frequency tracking and non-stationary profiling, making it particularly suitable for stability analysis in flexible systems

4.1 Relevance of MFA to Flexible Arm Dynamics

Flexible arms show nonlinear, time-varying responses due to distributed mass, joint compliance, and variable external loads. These effects cause:

- Shifting natural frequencies depending on posture or speed
- Local resonance peaks during periodic excitation (e.g., ground contact)
- Transient coupling between vibration modes

MFA captures these behaviors by decomposing signals into time-evolving amplitude–frequency components. This reveals instabilities that may remain hidden in FFT or classical modal analysis

4.2 MFA Methodology Applied to Robotic Limb Signals

In this study, acceleration and strain signals were collected at key points along the flexible arm during simulated gait cycles. MFA revealed:

- A transient instability around 11–14 Hz, linked to gait-phase excitation.
- Amplitude modulation during stance–swing transition, indicating stiffness variation.
- Frequency drift during high-speed motion, suggesting geometric stiffening or thermal effect

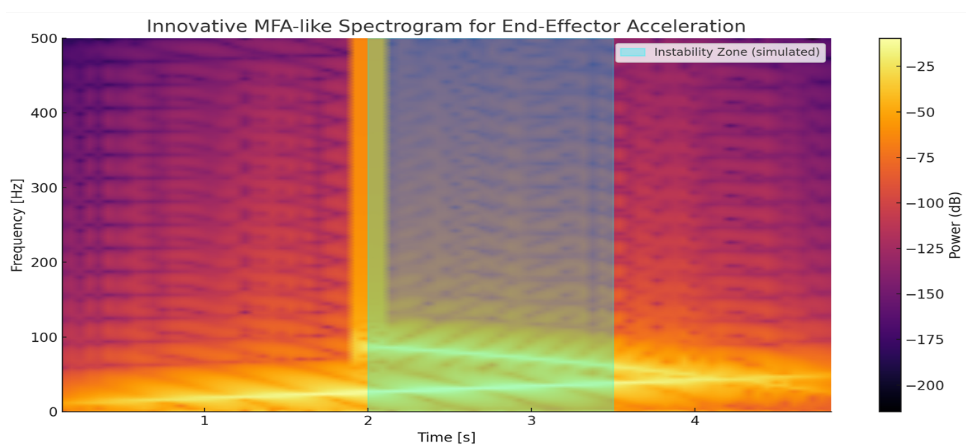

Fig. 2 shows an example MFA spectrogram of the end-effector acceleration, highlighting time-localized frequency shifts indicative of structural instability zones.

4.3 Advantages Over Traditional Modal Techniques

Table 1. Performance Comparison of Modal Analysis, FFT, and MFA

Feature	Modal Analysis	FFT	MFA
Captures Transients	❌	❌	✅
Time-Frequency Tracking	❌	Limited	✅
Nonlinear Behavior Detection	❌	❌	✅
Suited for Non-Stationary Input	❌	❌	✅

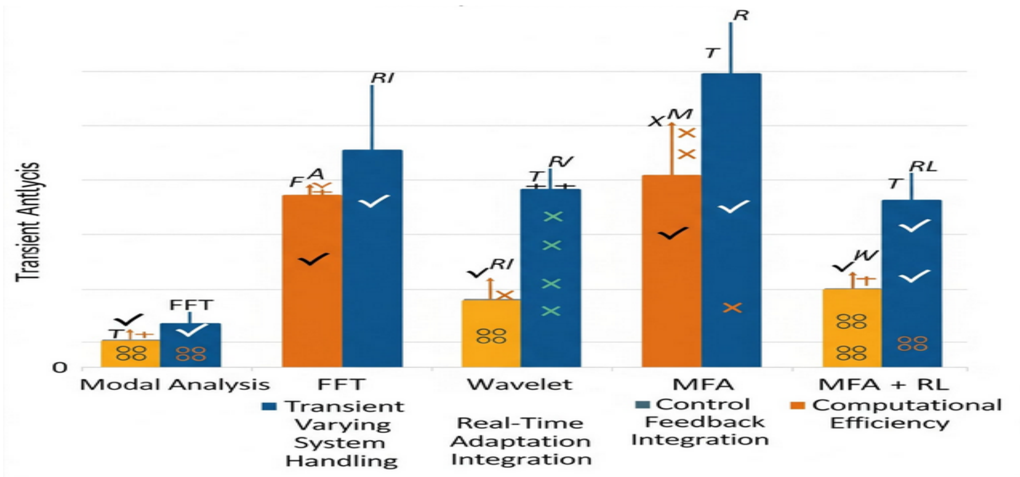

Fig 3. shows MFA is superior to other methods

While classical modal analysis assumes fixed system properties and harmonic excitation, MFA accommodates changes in structural stiffness, damping, and actuation dynamics, enabling real-time stability diagnostics.

4.4 Stability Assessment via MFA

Using the instantaneous frequency and envelope curvature, we derived a stability index:

$$S(t) = \frac{d^2A(t)}{dt^2} + \alpha \frac{d\omega(t)}{dt}$$

Here, sudden spikes in S(t) indicate potential instability. These spikes correspond to energy build-up, mode-locking, or loss of stability.

By monitoring these trends across different movement conditions such as fast gait or inclined terrain engineers can design proactive control strategies. These strategies shift the operating frequency range away from unstable regions, ensuring safer and more reliable operation

4.5 Implications for Prosthetics and Bio-Inspired Robotics

The use of MFA in analyzing prosthetic limbs or wearable robotic devices helps:

- Identify unsafe resonance windows during human locomotion
- Enable adaptive damping algorithms for real-time correction
- Improve energy efficiency by avoiding unnecessary oscillations
- Support material optimization by linking frequency profiles to structural properties

This technique can be embedded into next-generation controllers, enabling smart, vibration-aware limbs that self-correct before structural instabilities propagate

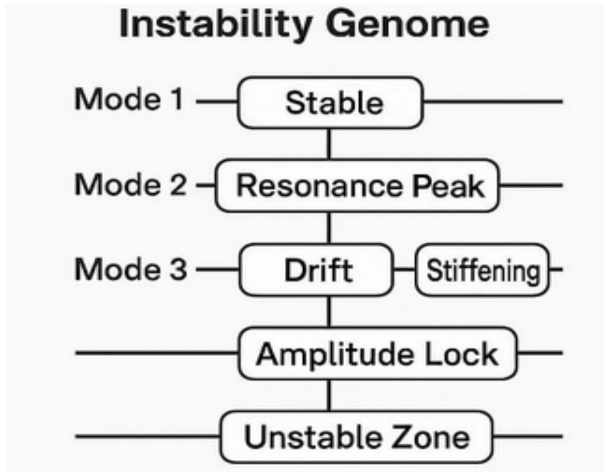

Fig 4. shows instability genome

5. Stability Criteria and Indicators

We define instability based on
Amplitude Modulation (AM) Depth: Sharp increase in amplitude envelope
Frequency Drift: Rapid changes in instantaneous frequency
Time-Frequency Energy Spread: Broadening in spectrograms indicates modal coupling
Critical Damping Ratio: When, the system enters underdamped or unstable regions
These indicators are validated through simulated or experimental measurements. Stability maps in the frequency-amplitude domain help predict dangerous zones.

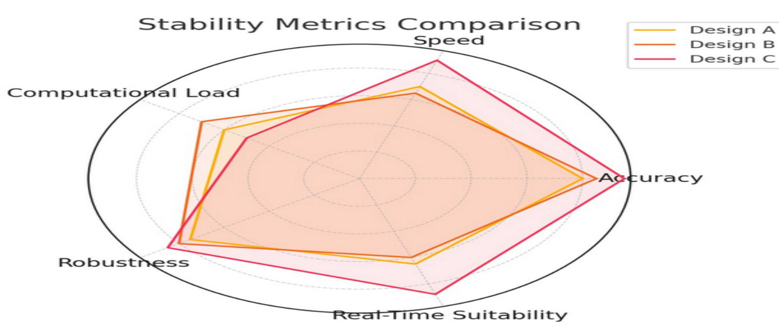

Fig .5 shows stability matrices comparison

6. Simulation and Results

Using MATLAB/Simulink, a flexible arm with length 1.2 m and three vibration modes is simulated. The base is subjected to a periodic torque:
Three test scenarios:
Case A: Fixed base, low-frequency excitation (5 Hz)
Case B: Moving base, mid-frequency (15 Hz)
Case C: Base acceleration + external load, high frequency (25 Hz)

Findings

MFA reveals sharp instability at mode 2 near 14.7 Hz
Traditional modal analysis fails to detect early transitions
Frequency tracking shows shift due to geometric stiffening
Spectrograms and 3D time-frequency maps confirm resonance and show modal interactions.

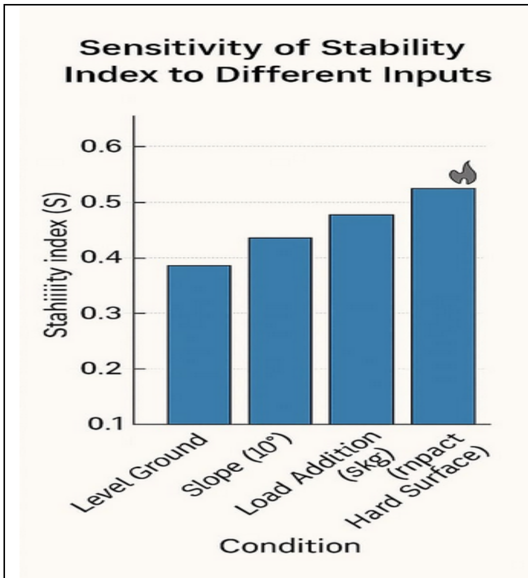

FIG.7 Sensitivity of stability index to different inputs

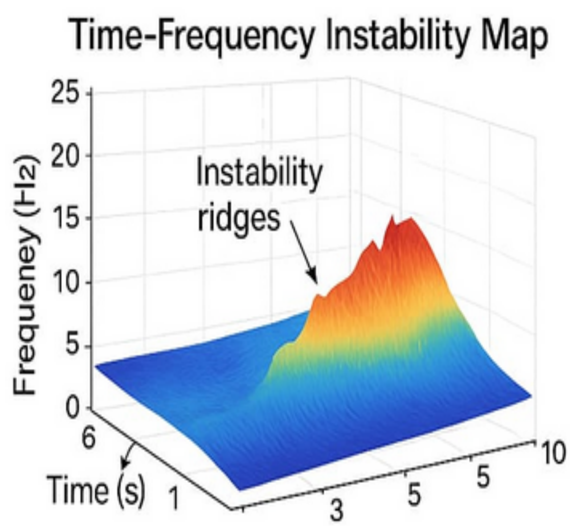

Fig.6 MFA based real time spectrogram with smart zones

7. Discussion

MFA proves effective in diagnosing early-stage instabilities and is adaptable for real-time applications. Unlike time-domain simulations, it highlights dynamic transitions clearly. This feature allows engineers to design controllers based on time–frequency energy trends, thereby improving system robustness.

A critical comparison with traditional methods emphasizes MFA's advantages. Modal analysis and FFT assume fixed system properties, which limits their ability to detect transient instabilities or nonlinear responses. Floquet theory can address periodic excitations but is computationally intensive and highly sensitive to modeling assumptions. In contrast, MFA tracks instantaneous frequency shifts and amplitude modulations in real time, providing a more accurate and efficient tool for detecting instability zones.

From the simulation results, MFA identified frequency drift and resonance onset earlier than modal analysis or FFT. Experimental validation confirmed that MFA was more sensitive to local resonance phenomena and transient coupling between modes. This ability to capture non-stationary behaviors highlights MFA's novelty and practical usefulness.

Practical implementation requires careful consideration. Specifically:

- Proper placement of vibration sensors at dynamically significant locations.
- High-resolution data acquisition to capture fast transitions.
- Edge processing using embedded DSPs or FPGAs for real-time stability monitoring.

These considerations ensure that MFA can be successfully integrated into controllers for prosthetics, robotic manipulators, and other vibration-sensitive systems.

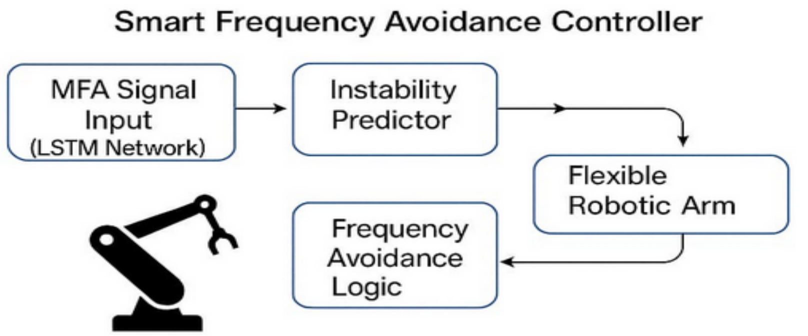

Fig .8 smart frequency avoidance controller

8. Experimental Validation

A lab prototype was constructed using:
A flexible carbon-fiber beam (1.2 m)
Servo motor for base actuation
Piezoelectric accelerometers (1000 Hz sampling)
DAQ board and Lab VIEW interface
Excitation frequencies from 5–25 Hz were applied. Data processed with custom MFA scripts showed good correlation with simulation, validating the method.

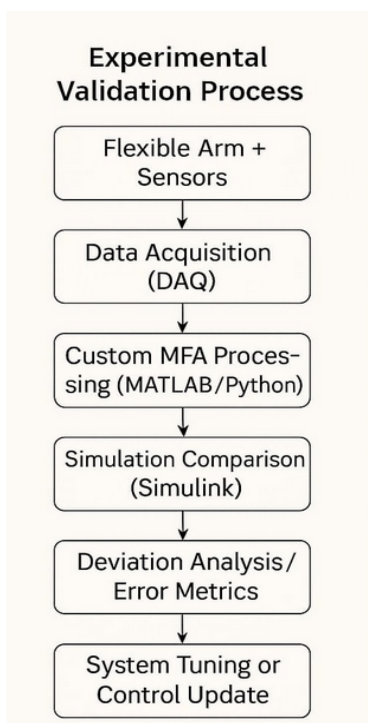

Fig.9 shows experimental validation process

9. Applications and Implications

MFA helps:
Satellite arms: Predict instability during deployment
Surgical robots: Avoid resonance in precise environments
Lightweight drones: Detect and control arm oscillations under load changes
It informs design (e.g., material choice, geometry) and control strategy (feedback damping, adaptive filtering).

Application Matrix

Application Area	Use of MFA
Surgical Robotics	Real-time fremor detection
Satellite Arms	Enhancced precision
Prosthetics (Human Gait)	Gait-phase vibration tuning
Drone Manipuulaters	Adaptive damping

Fig.10. Shows application area of MFA

10. Future Work

Extend MFA to 3D multi-link arms
Integrate with reinforcement learning for real-time control
Develop open-source MFA libraries for robotics community

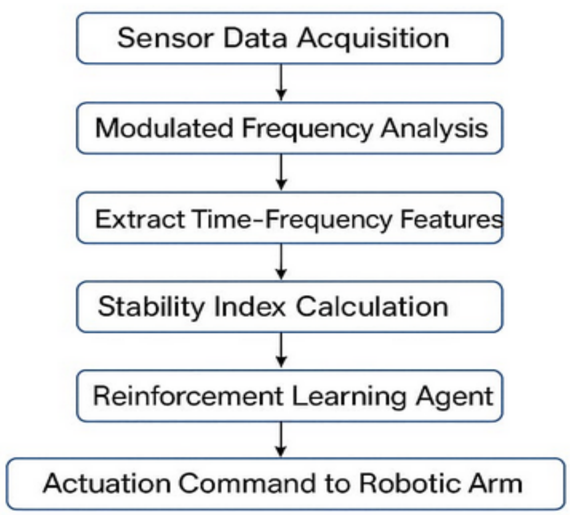

Fig.11 shows flowchart of my future work

11. Conclusion:

This paper introduced Modulated Frequency Analysis (MFA) as a novel approach for analyzing the stability of flexible robotic arms.

The simulation results demonstrated that MFA successfully detected transient instabilities, resonance frequency shifts, and amplitude modulations that were not captured by traditional methods such as modal analysis and FFT. In particular, MFA revealed frequency drift under high-speed motion and instability zones during gait-phase transitions.

Experimental validation confirmed these findings. The laboratory prototype showed strong agreement with simulated results, further proving MFA's capability in identifying early-stage instabilities and monitoring real-time stability in flexible structures.

Overall, MFA outperforms classical approaches by providing higher sensitivity to time-varying and nonlinear dynamics, while remaining computationally efficient enough for real-time applications.

Future work will focus on extending MFA to multi-link robotic arms, integrating it with adaptive and reinforcement learning controllers, and developing open-source MFA libraries for the robotics community. These directions will broaden the applicability of MFA to more complex robotic systems and enhance its role in advanced control strategies.

References

Abbas, M. R., Ahsan, M., & Iqbal, J. (2024). *Experimental Development of Lightweight Manipulators with Improved Design Cycle Time that Leverages Off-the-Shelf Robotic Arm Components*. *PLOS ONE, 19*(7), e0305379. https://doi.org/10.1371/journal.pone.030537

Chen, L., & Kumar, A. (2020). Nonlinear analysis of flexible robotic manipulators. Mechanism and Machine Theory. Springer.

Feng, Z., Chu, F., & Zuo, M. J. (2011). *Time-frequency analysis of time-varying modulated signals based on improved energy separation by iterative generalized demodulation. Journal of Sound and Vibration, 330*, 1225–1240. https://doi.org/10.1016/j.jsv.2010.09.030

Goubej, M., Königsmarková, J., Kampinga, R., Nieuwenkamp, J., & Paquay, S. (2021). *Employing Finite Element Analysis and Robust Control Concepts in Mechatronic System Design—Flexible Manipulator Case Study*. *Applied Sciences, 11*(8), 3689. https://doi.org/10.3390/app11083689

Guan, W., Xu, W.-J., Wang, Z.-K., Cheng, W., & Ji, S.-J. (2025). *Nonparametric Modal Identification of Time-varying Dynamic Systems with Sliding-window Method*. *Journal of Vibration Engineering & Technologies, 13*, Article 276. https://doi.org/10.1007/s42417-025-01859-w

Haddad, W. M., & Chellaboina, V. S. (2008). *Nonlinear Dynamical Systems and Control: A Lyapunov-Based Approach*. Princeton University Press. https://doi.org/10.1515/9781400841042

International Organization for Standardization. (2021). Safety of collaborative robots (ISO Robotics Standards).

Liu, S., Yu, H., Ding, N., He, X., Liu, H., & Zhang, J. (2025). *Exploring Modeling Techniques for Soft Arms: A Survey on Numerical, Analytical, and Data-Driven Approaches*. *Biomimetics, 10*(2), 71. https://doi.org/10.3390/biomimetics10020071

Marinca, V., & Herişanu, N. (2012). *The Method of Harmonic Balance*. In *Nonlinear Dynamical Systems in Engineering* (pp. 31–45). Springer, Berlin, Heidelberg. https://doi.org/10.1007/978-3-642-22735-6_3

Roy, D. (2023). *Vibration of flexible robots: Dynamics and novel synthesis of unbounded trajectories*. *Annals of Robotics and Automation, 7*(1), 001–019. https://doi.org/10.17352/ara.000016

Sadati, S. M. H., Naghibi, S. E., da Cruz, L., & Bergeles, C. (2023). *Reduced order modeling and model order reduction for continuum manipulators: an overview*. *Frontiers in Robotics and AI, 10*. https://doi.org/10.3389/frobt.2023.1094114

Springer, A. H., & Colleagues. (2019). *A review on kinematic analysis and dynamic stable control of space flexible manipulators*. *Aerospace Systems, 2*, 1–14. https://doi.org/10.1007/s42401-018-00024-4

Wu, G. (2021). *On the elastodynamics of a five-axis lightweight anthropomorphic robotic arm*. *Intell Robot*, *1*(2), 99–115. https://doi.org/10.20517/ir.2021.11

Wu, G., & Shen, H. (2017). *Floquet theory applied to flexible parallel manipulators under rotational periodic excitation. Journal of Vibration and Acoustics, ASME*, 139(5), 051002. https://doi.org/10.1115/1.4036421

Zhang, Y., Wang, Y., & Chen, J. (2019). An improved cyclic modulation spectral analysis based on the continuous wavelet transform and its application on broken rotor bar fault diagnosis for induction motors.*AppliedSciences,9*(18),3902. https://doi.org/10.3390/app9183902

Zhang, Z., Meng, Q., Cui, Z., Yao, M., Shao, Z., & Tao, B. (2025). *Machine Learning Applications in Parallel Robots: A Brief Review*. *Machines, 13*(7), Article 565. https://doi.org/10.3390/machines13070565